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НЕОКЛАСИЧНИЙ ДІАМАГНІТНИЙ СТРУМ У ПЛАЗМІ ТОКАМАКА З ЕЛЕКТРОННО-ЦИКЛОТРОННИМ НАГРІВАННЯМ

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Р е з ю м е

Нагрівання методом електронного циклотронного резонансу в токамаках приводить до генерації надтеплових електронів, що "утікають" у просторі швидкостей вздовж характеристик квазілінійної дифузії. Дрейфовий рух в магнітному полі токамака викликає тороїдальну прецесію цих гарячих захоплених електронів, генеруючи таким чином безіндукційний струм. У рамках напіваналітичної моделі обчислено напрямок, величину та профіль цього струму. Для типових рівнів ВЧ-потужності у токамаці RTP (Голландія) щільність струму сягає $j_{||} \sim 140$ A/cm^2, що може призвести до суттєвого "сплощення" профілю q і, як наслідок, до стабілізації пилкоподібних коливань.

НЕОКЛАССИЧЕСКИЙ ДИАМАГНИТНЫЙ ТОК В ПЛАЗМЕ ТОКАМАКА С ЭЛЕКТРОННО-ЦИКЛОТРОННЫМ НАГРЕВОМ

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Нагрев методом электронного циклотронного резонанса в токамаках приводит к генерации сверхтепловых электронов, "убегающих" в пространстве скоростей вдоль характеристик квазилинейной диффузии. Дрейфовое движение в магнитном поле токамака вызывает тороидальную прецессию этих горячих захваченных электронов, генерируя таким образом безиндукционный ток. В рамках полуаналитической модели вычислены направление, величина и профиль этого тока. Для типичных уровней ВЧ-мощности в токамаке RTP (Голландия) плотность тока достигает $j_{||} \sim 140$ A/см^2, что может привести к существенному "уплощению" профиля q и, как следствие, к стабилизации пилообразных колебаний.

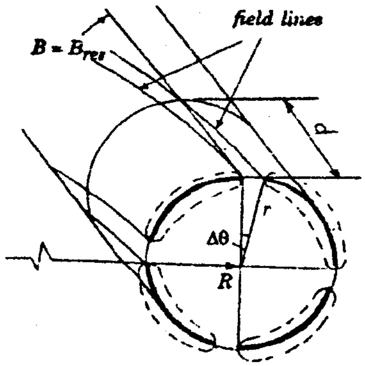

Fig. 3. Poloidal cross-section of the magnetic surface with $q = 0.8$

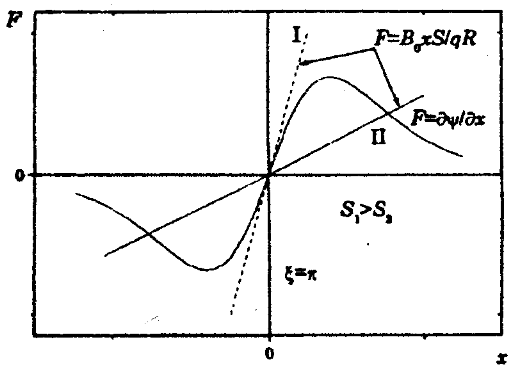

Fig. 4. Singular points of Eq. (19) at $\xi = \pi$. Case 1: $S > \frac{qR}{B_0}\frac{\partial^2\tilde{\psi}}{\partial x^2}(0, \pi)$. Case 2: $S < \frac{qR}{B_0}\frac{\partial^2\tilde{\psi}}{\partial x^2}(0, \pi)$

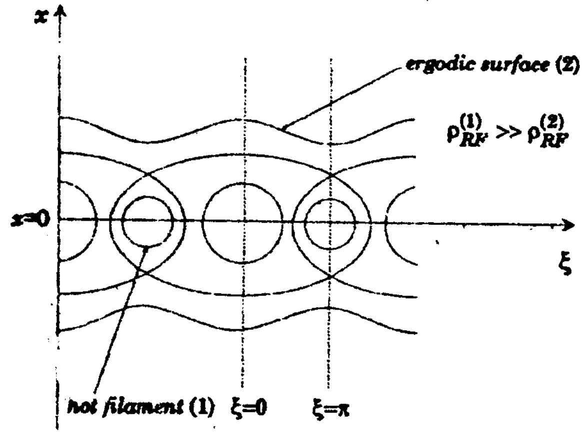

Fig. 5. Contours ψ = const of Eq. (19) for the Case 2 of Fig. 4

Here, $x = r - r_s$ is the distance from the surface, $S = r_s q'/q$ is magnetic shear, $\xi = m\theta - n\zeta$, ζ is the toroidal angle, and $q_s = m/n$. The perturbation amplitude $\tilde{\psi}(x) \to 0$ for $x \to \pm\infty$ and could be chosen in the form $\tilde{\psi}(x) = A\exp(-x^2/b^2)$. The usual constant $\tilde{\psi}$ approximation [9], which is widely used in the literature, is valid if $b \gg w$, with w being the magnetic island width. This approximation breaks down for low shear. Indeed, the singular points of Eq. (19) are defined by (see also Ref. [10]):

$$\begin{cases} -\dfrac{xSB_0}{qR} - \dfrac{2x}{b^2}Ae^{-x^2/b^2}\cos\xi = 0, \\ \sin\xi = 0. \end{cases} \tag{20}$$

For $\xi = \pi$, two cases are possible depending on the shear value, as shown in Fig. 4. Thus, when shear decreases (precessional current, described above, builds up), the "conventional" hyperbolic point ($x = 0$, $\xi = \pi$) bifurcates to three singular points, two of which ($x = 0$) are hyperbolic and one ($x = 0$) is elliptic (Fig. 5).

Now the preferential heating between highlighted regions in Fig. 3 becomes possible, because a new "X-point" contains its own closed flux tubes, which turn in the opposite direction to islands formed at the places of highlighted regions. As a result, thermal instability develops.

It should be noted that, as a result of abrupt redistribution of the absorbed ECH power on the rational surface, the ratio of the absorbed power density inside a filament to that on the ergodic surface is given by $\rho_{RF}^{(1)}/\rho_{RF}^{(2)} = v_{QL}^{(1)}/v_{QL}^{(2)} = 2\pi qR/md \sim 10$ for $m \le 10$, $q \le 1$, $R \sim 1$ m, $d \sim 10$ cm. If we also assume that thermal conductivity inside the filament is lower than that on the ergodic surface, then a large temperature variation between filaments and ergodic zones becomes possible, as observed in experiment [2].

Of course, the above speculations should be supported by rigorous treatment, including a self-consistent solution of the thermal balance and Grad — Shafranov equations, where both current and temperature (pressure) are the functions of a helical flux in the form (19). It is a separate problem, which should be addressed to the future work.

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where $\Gamma(x)$ is the gamma function and the second equality is valid for $\epsilon << 1$.

In Fig. 1, the resultant hot electron current density profile is shown, calculated for the model profiles $n_e(r) = n_0(1 - r^2/a^2)^{1/2}$, $T_e(r) = T_0(1 - r^2/a^2)^2$ and parameters close to those in ECR heating experiments in the RTP tokamak (Table) [2].

3. Flattening of the q-profile and possibility of the current/temperature filamentation

The non-inductive hot electron current will modify the initial current density and resulting q-profile. Fig. 2 shows the results for the initial current density profile $j = j_0(1 - r^2/a^2)^\alpha$, where $j_0 \approx 400$ A/cm^2 is chosen to yield a fixed edge safety factor $q_a = 3.2$ and $\alpha = 3$ is used to give the total current $I_p = 120$ kA. The q-profile is modified considerably, exhibiting a significant flattening. Such a flattening results in suppression of the sawtooth activity [8], which was observed in experiment [2].

Another interesting consequence of the q-profile flattening is the possibility of generation of the hot current filaments, which was the most intriguing phenomenon observed in [2]. It is not the aim of the present work to develop the model of such a filamentation, but to discuss briefly the reasons for filaments generation in plasma with the extended region of ultralow magnetic shear in the central core.

Let the tokamak plasma be heated by the microwave beam with finite toroidal width d and frequency ω corresponding to the electron cyclotron resonance at the magnetic axis (Fig. 3). Then electrons on the magnetic surface experience a quasilinear diffusion in velocity space with the coefficient $D = D_0\gamma$, where $D_0 = \nu_{QL} v_{Te}^2 \delta(\theta - \pi/2)$ is given in Eq. (8) and the geometrical factor γ accounts for a finite toroidal width of the beam and is strongly dependent on the safety factor q of the magnetic surface.

If q is irrational, then a circulating electron covers the magnetic surface ergodically and $\gamma \sim \Delta\theta/2\pi = d/2\pi qR$ (see Fig. 3). The situation changes considerably when the surface is rational. Then magnetic field lines rejoin after some transits of the torus, so that electrons on that collection of

Plasma parameters

Major radius, R_0	1 m
Minor radius, a	20 cm
Electron density on the axis, n_0	$2.5 \cdot 10^{13}$ cm^{-3}
Electron temperature on the axis, T_0	2.5 keV
Magnetic field on the axis, B_0	2.2 T
Plasma current, I_p	120 kA
RF power deposition profile parameter, Δ	4 cm
RF power density on the axis, p^0_{RF}	10 W/cm^3

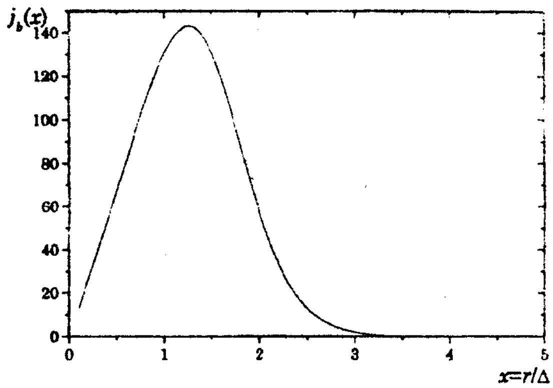

Fig. 1. Flux surface averaged hot electron current density for a semi-analytic model and parameters given in Table

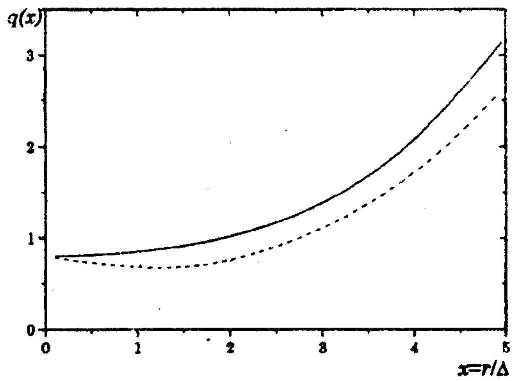

Fig. 2. Effect of the hot electron current on the q-profile. The full line is calculated using the current density profile $j(r) = j(0)(1 - (r/a)^2)^3$. The addition of the noninductive component leads to a flattening of the q-profile with $dq/dr = 0$

field lines, which is highlighted in Fig. 3, intersect the resonance surface $\omega = \omega_{ce}$ outside the beam location, which means that $\gamma = D = 0$ for these electrons. Thus, on a rational q-surface, the ECH power is absorbed on a few field lines only (between highlighted regions). These regions have a potential to become X-points of the reconnected poloidal magnetic flux. However, if magnetic shear is finite, it is impossible to heat an X-point (and sustain a sufficient current perturbation $\delta j/j_0 \sim \frac{3}{2}\delta T_e / T_0$) because of its topology. The situation changes in the case of low shear.

Indeed, in the vicinity of the rational surface, the magnetic field can be described by the effective helical flux function

$$\psi = -\frac{x^2 S}{2qR} B_0 + \tilde{\psi}(x)\cos\xi. \tag{19}$$

then reads

$$f_h = \frac{4n_e}{\sqrt{\pi}} \frac{\int\limits_{u_*}^{\infty} u^2 \exp(-u^2)\, du}{\int\limits_{u_*}^{u_{\max}} u^2 \exp(2u_*^3/u)\, du} \times$$

$$\times \exp\left(\frac{2u_*^3}{u}\right) (H(u_{\max} - u) - H(u - u_*)), \qquad (11)$$

where $u_{\max} = (2mc^2 / T_e)^{1/4}$ and the normalization constant is obtained from the conservation of the total number of particles.

The last step is to relate u_* with the absorbed power density:

$$\langle \rho_{RF} \rangle = \frac{m}{2} \langle \int d^3 v\, v_\perp^2\, Q(f) \rangle \approx \frac{m}{2} \nu_{QL} v_{Te}^2 n_h,$$

from which we obtain the transcendental equation for u_*:

$$\frac{2}{\sqrt{\pi}} \frac{\nu_0(r)\, n_e(r)\, T_e(r)}{u_*^3} \int\limits_{u_*}^{\infty} u^2 e^{-u^2} du \approx \rho_{RF}^0 \exp\left(-\frac{r^2}{\Delta^2}\right), \qquad (12)$$

where the Gaussian deposition profile was assumed. In the next section, the model distribution function (11), (12) is used for calculation of the precessional current carried by hot electrons.

2. Diamagnetic current by strong ECR heating

From the steady state drift kinetic equation [6] within a thin orbit approximation in an axisymmetric tokamak with a circular cross-section, one can derive a flux surface averaged expression for the bootstrap current j_B of fast particles (see Ref. [7] for the derivation):

$$j_B = \frac{q}{2\pi} \int\limits_0^{2\pi} \frac{\pi}{h} \sum_\sigma \sigma \int\limits_0^{\infty} v^3 \int\limits_0^{h} f_1\, h\, d\lambda\, dv\, d\theta, \qquad (13)$$

where

$$f_1 = -\frac{m c v_\parallel}{qB_\theta} \frac{\partial f_0}{\partial r} + g(v, \lambda, r). \qquad (14)$$

Here, q and m are the particle charge and mass, respectively, $\sigma = v_\parallel / |v_\parallel|$, $h = 1 + \epsilon \cos\theta$, $\epsilon = r/R$, r is the plasma radius, R is the major radius of the tokamak, and B_θ is the poloidal magnetic field component. The integral in Eq. (13) is over the poloidal angle θ, total velocity v and $\lambda = hv_\perp^2 / v^2$, with $v_\perp$ denoting the perpendicular component of the velocity with respect to the magnetic field. Expression (13) is found by expanding the particle distribution $f = f_h(v, \lambda, r) + f_1(v, \lambda, \theta, r)$ to the first order in $\rho_\theta / L \ll 1$ and by making the approximation $\nu_{QL} \tau_b \ll 1$, where ρ_θ is the poloidal Larmor of hot electrons, L is the gradient scale length for the plasma and wave parameters, τ_b is the bounce time of electron banana orbits. Moreover, to obtain Eqs. (13) and (14), the collisional and RF operators have been assumed to be independent of the sign of the parallel velocity component $v_\parallel (k_\parallel = 0)$. On this assumption, one can also conclude that the function $g(v, \lambda, r)$, which is independent of θ and σ, vanishes for trapped particles ($\lambda > 1 - \epsilon$).

Consistently with our model Eq. (11) for f_h, the trapped electron contribution to the bootstrap current is written as

$$j_B = -\frac{mc}{B_\theta} \int\limits_0^{2\pi} d\theta \int\limits_{v_*}^{v_{\max}} dv \int\limits_{1-\epsilon}^{h} d\lambda \sqrt{1 - \lambda/h}\, \frac{\partial f_h}{\partial r} v^4, \qquad (15)$$

where the integral over λ covers only the trapped particles.

With a new pitch angle variable λ instead of ξ, the full distribution function of hot electrons takes the form

$$f_h(v, \lambda, r) = C\, \delta(\lambda - 1) f_h(v, r), \qquad (16)$$

where $f_h(v, r)$ is given by Eq. (11) and the normalization constant C is determined by the condition

$$\frac{C}{2\pi} \int\limits_{-\pi/2}^{\pi/2} d\theta \sum_\sigma \int\limits_0^{h} \frac{\pi}{h} \frac{\delta(\lambda - 1)}{h\sqrt{1 - \lambda/h}} d\lambda = 1. \qquad (17)$$

By inserting Eq. (16) in (15) and averaging over θ, the trapped electron contribution to the bootstrap current becomes

$$j_B = -\epsilon \frac{mc}{B_\theta} \frac{\int\limits_{-\pi/2}^{\pi/2} \sqrt{\frac{\cos\theta}{h}}\, d\theta}{\int\limits_{-\pi/2}^{\pi/2} \frac{1}{h} \sqrt{\frac{h}{\cos\theta}}\, d\theta} \int\limits_{v_*}^{v_{\max}} \frac{\partial f_h}{\partial r} v^4\, dv \approx$$

$$\approx -\epsilon \frac{\Gamma(5/4)\, mc}{\Gamma(3/4)\, B_\theta} \int\limits_{v_*}^{v_{\max}} \frac{\partial f_h}{\partial r} v^4\, dv, \qquad (18)$$

$$\nu_0 = \frac{\sqrt{2}\,\pi n_0 e^4 \ln \Lambda}{\sqrt{m_e}\, T_e^{3/2}}, \tag{5}$$

$$D = \sum_l \frac{\pi e^2}{8 m_e^2} |E_-|^2 J_{l-1}^2 (k_\perp \rho)\, \delta(\omega - l\omega_{ce}). \tag{6}$$

Here, $v_{Te} = (2T_e / m_e)^{1/2}$ is the electron thermal speed, $\xi = v_\parallel / v$ is the pitch angle, E_- represents the right-handed circulary polarized electric field of the ordinary wave, D is the quasilinear velocity space diffusion coefficient driven by cyclotron waves at the resonant location, J_{l-1} is the Bessel function of the first kind of order $l - 1$, $k_\perp$ is the wave vector perpendicular to the equilibrium magnetic field, and ρ is the gyroradius. The delta function in D requires that the particles should be resonant somewhere along their orbits to have a nonvanishing D and is related to the absorbed wave power. Here after, we consider a fundamental resonance ($l = 1$), so that D is energy independent for millimeter waves ($k_\perp \rho << 1$). It should be noted that the energy dependence in (6) could appear due to relativistic mass in ω_{ce}. Together with the finiteness of the Larmor radius, this effect defines the limiting energy for runaway (see below).

The bounce-averaged quasilinear diffusion operator takes the form [3]:

$$\{Q\} = \frac{1}{v^2} \frac{\partial}{\partial v}\bigg|_{\xi_0} \left(D_b v^2 \frac{\partial f}{\partial v}\bigg|_y \right) - \frac{1}{v} \frac{\partial}{\partial \xi_0}\bigg|_v \left(D_b \frac{\xi_0^2 - \epsilon}{\xi_0} \frac{\partial f}{\partial v}\bigg|_y \right), \tag{7}$$

where

$$D_b \equiv \{D\} = \frac{\pi}{2} \frac{\omega}{\epsilon} \left(\frac{cE_-}{B_0} \right)^2,$$

$$\xi_0 = \frac{v_\parallel(\theta = 0)}{v}.$$

On axes, a resonance was assumed ($\theta_{res} = \pm \pi/2$), and $y = v_{\parallel res}$ is the parallel velocity at the resonance location. Performing a bounce averaging of the pinch-angle scattering operator, one could obtain the Fokker — Planck equation in the form:

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left(v^2 \nu_0 \left(\frac{v_{Te}}{v} \right)^3 \left(vf + \frac{v_{Te}^2}{2} \frac{\partial f}{\partial v} \right) \right) + \frac{1 + z_{\rm eff}}{2\{v/v_\parallel\}} \nu_0 \left(\frac{v_{Te}}{v} \right)^3 \frac{1}{\xi_0} \frac{\partial}{\partial \xi_0} \left(\left\{ \frac{v_\parallel}{v} \right\} \frac{1 - \xi_0^2}{\xi_0} \frac{\partial f}{\partial \xi_0} \right) +$$
$$+ \nu_{QL} v_{Te}^2 \left[\frac{1}{v^2} \frac{\partial}{\partial v}\bigg|_{\xi_0} \left(v^2 \frac{\partial f}{\partial v}\bigg|_y \right) - \frac{1}{v} \frac{\partial}{\partial \xi_0}\bigg|_v \left(\frac{\xi_0^2 - \epsilon}{\xi_0} \frac{\partial f}{\partial v}\bigg|_y \right) \right]. \tag{8}$$

Strictly speaking, determination of the hot electron distribution function requires to solve the two-dimensional bounce-averaged Fokker — Planck equation (8). Note, however, that runaway electrons tend to form a "jet" in velocity space along the RF diffusion path (see Intriduction). Assuming $f(\xi, v) = f(v)\, \delta(\xi - \sqrt{\epsilon})$, we can evaluate $f(v)$ by pith-angle integration of Eq. (8). This procedure is widely used in the theory of minority ion cyclotron heating [4]. Then one could obtain for the stationary distribution

$$\frac{1}{v^2} \frac{\partial}{\partial v} \left[v^2 \nu_0 \left(\frac{v_{Te}}{v} \right)^3 \left(vf + \frac{v_{Te}^2}{2} \frac{\partial f}{\partial v} \right) + \nu_{QL} v_{Te}^2 v^2 \frac{\partial f}{\partial v} \right] = 0. \tag{9}$$

The solution then reads

$$f = C \exp \left(-\int_{u_*}^{u} \frac{\nu_0\, u\, du}{\nu_{QL} u^3 + \nu_0/2} \right), \tag{10}$$

where $u = v / v_{Te}$, and $u_* \equiv (\nu_0 / 2\nu_{QL})^{1/3} > 1$ defines the boundary between the Maxwellian isotropic background ($u < u_*$) and highly anisotropic hot electron ($u > u_*$) populations. It follows from (10) that, for $u \to \infty, f \to$ const indicating runaway. Thus, it is necessary to identify a physical mechanism which cuts off the runaway distribution at high energies. There are two possibilities: 1) finite gyroradius cut off $J_0(k_\perp \rho) = 0$; 2) relativistic resonance detuning [5], which gives the maximum energy $\mathcal{E}_{max} \sim (2\varepsilon_{\parallel 0} mc^2)^{1/2}$, where $\varepsilon_{\parallel 0} \sim T_e$ is the initial energy of longitudinal motion. For typical parameters, the second mechanism gives a lower limiting energy and will be used hereafter. The distribution function

NEOCLASSICAL DIAMAGNETIC CURRENT IN ELECTRON CYCLOTRON HEATED TOKAMAK PLASMA

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Electron cyclotron resonance (ECR) heating in a tokamak can produce superthermal runaway electrons having their turning points located at the cyclotron resonance surface (so-called "sloshing" electrons). The drift motion causes these trapped hot electrons to precess around a torus, thus forming a hot electron current. A semi-analytic model is used to calculate the direction, magnitude, and profile of this current. For a typical high power central ECR heatin a hot electron current density $j_{\|} \sim 140\ \mathrm{A/cm^2}$ is found which is peaked at a radius ~ RF power deposition where it enhances the local current density. The effect of the hot electron current is to flatten the q-profile within the RF power deposition region. If this radius is comparable with the $q = 1$ radius, this results in sawtooth stabilization. Moreover, low magnetic shear is favourable for formation of hot current filaments, which was observed in experiment.

Introduction

As is well known, the MHD stable operation of tokamak discharges requires active control over the current density profile [1]. There are several possibilities for the current profile modification:

1) off-axis current drive with injection of neutral beams, electron cyclotron waves, and/or lower hybrid waves with finite $k_{\|}$ (longitudinal mode number);

2) discharge with high bootstrap current fraction achived by the pellet injection into the central core.

In the present work, an alternative possibility is considered, which is based on the generation of the hot, highly anisotropic electron population during on-axis electron cyclotron heating with $k_{\|} = 0$.

For a sufficiently high RF power, a significant fraction of electrons runs away along the RF diffusion path in velocity space. The equation for the RF diffusion characteristic reads

$$\varepsilon - \mu\omega = \text{const} \approx 0, \quad \text{for } \varepsilon >> T_e, \tag{1}$$

where ε, μ are the particle energy and magnetic moment, respectively, ω is the wave frequency, T_e is the temperature of background electrons, which are assumed to be isotropic. According to (1), the distribution function of runaway electrons takes the form $f(\varepsilon, \mu) = \delta(\varepsilon - \mu\omega) F(\varepsilon)$, which means that banana "tips" of these electrons accumulate on the resonance surface $\omega = \omega_{ce}$, where ω_{ce} is the cyclotron frequency.

In the limit of a negligible poloidal Larmor radius, parallel motions on the inner and outer branches of the banana orbit exactly cancel in one orbital period. However, with a finite poloidal Larmor radius (a few millimeters for hot electrons considered below), these motions do not longer cancel and, as is well known, the orbit as a whole precesses around a torus. Considering the whole ensemble of heated runaway electrons, this motion gives rise to a net diamagnetic current of hot electrons in the direction having the same sense as the plasma current.

In the next section, the function $F(\varepsilon)$ is determined by solution of the Fokker — Planck equation integrated over the pitch angle. This model function is then used in Sec. 2 to calculate precessional current for a particular set of parameters close to ECR heating experiments in RTP tokamak [2]. The effect of this current on the q-profile is considered in Sec. 3, where the possibility of the current/temperature filaments generation is discussed briefly.

1. Energy distribution of hot electrons

The Fokker — Planck equation for electrons has the form:

$$\partial f / \partial t = C(f) + Q(f), \tag{2}$$

where C and Q represent the Coulomb collision operator and quasilinear scattering operator, respectively:

$$C(f) = \frac{1}{2}\nu_0 \left(\frac{v_{Te}}{v}\right)^3 (1 + Z_{\text{eff}}) \frac{\partial}{\partial \xi}(1 - \xi^2)\frac{\partial f}{\partial \xi} +$$

$$+ \frac{1}{v^2}\frac{\partial}{\partial v} v^2 \nu_0 \left(\frac{v_{Te}}{v}\right)^3 \left(vf + \frac{v_{Te}^2}{2}\frac{\partial f}{\partial v}\right), \tag{3}$$

$$Q(f) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp D \frac{\partial f}{\partial v_\perp}, \tag{4}$$

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