FEATURES OF NONLINEAR DRIFT WAVE INTERACTIONS NEAR MARGINAL STABILITY BOUNDARY IN PLASMAS

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The dynamics of nonlinear drift waves interactions is shown to be strongly modified when interaction energy strongly exceeds the eigen wave energies. The conditions of modified explosive instability saturation due to convective wave losses from the wave coupling region have been found. The possibility of solitons and other nonlinear wave structures formation during the nonlinear wave coupling in plasmas near marginal stability boundary is shown.

Density $n(r)$ and temperature $T(r)$ profiles in plasma of tokamaks often nearly satisfy conditions for marginal stability of strong reactive unstable modes such as the ion temperature gradient driven mode, or η_i-mode ($\eta_i = d\ln T/d\ln n$) [1]. Below the linear stability boundary there still exists rather high level of turbulence, so called "subcritical turbulence". In [2] the nonlinear explosive instability due to interaction between modes with positive and negative energy was proposed as the driving mechanism for the turbulence. As shown [3 — 5] the character of nonlinear wave interaction radically changes near the stability boundary. When all interacting modes are " zero-energy modes" characteristic nonlinear interaction time t_0 is the smallest [4,5]. Then mode energy exchange during nonlinear interaction greatly exceeds "own" energies of modes. Other important feature of nonlinear wave interaction near stability boundary is the following. The possibility of explosive instability does not depend on signs of "own" wave energies as well as on signs of interaction matrix elements if two or three waves from resonant triad are "zero-energy waves" [3,6]. In this paper we shall consider spatial-temporal evolution of resonantly interacting drift waves near marginal stability boundary. It will be shown that some coherent structures can be formed as a result of this interaction.

In slab geometry with all inhomogeneities (density, temperature and magnetic field) in the radial ($\hat{x}$)-direction we use the fluid model proposed in [7] governing ion and electron densities, electrostatic potential and ion temperature evolution. In this model electrons are assumed to be adiabatic and ions are assumed to move perpendicular to the external magnetic field in wave motion. Nonlinear coupling of η_i-modes is determined mostly by the nonlinear $\mathbf{E}\times\mathbf{B}$ drift (so called vector nonlinearity). Taking for simplicity $T_i = T_e$ after Fourier transformation the basic set of equations has the form ([7], (8a), (8b)):

$$\left(i\frac{\partial}{\partial t} + k_y A\right) T_k + k_y B\Phi_k =$$
$$= \frac{i}{2}\sum_{k_1,k_2} (\mathbf{k}_1\times\mathbf{k}_2)\cdot\mathbf{e}_{\parallel}\left[\frac{4}{3}\left(k_2^2 - k_1^2\right)\Phi^*_{k_1}\Phi^*_{k_2} + \right.$$
$$\left. + \frac{2}{3}\left(k_2^2 T^*_{k_1}\Phi^*_{k_2} - k_1^2\Phi^*_{k_1}T^*_{k_2}\right)\right], \tag{1}$$
$$\left(i\frac{\partial}{\partial t} + k_y D\right)\Phi_k + k_y C T_k =$$
$$= \frac{i}{2}\sum_{k_1,k_2}\frac{(\mathbf{k}_1\times\mathbf{k}_2)\cdot\mathbf{e}_{\parallel}}{1+k^2}\left[2\left(k_2^2 - k_1^2\right)\Phi^*_{k_1}\Phi^*_{k_2} + \right.$$
$$\left. + \left(k_2^2 T^*_{k_1}\Phi^*_{k_2} - k_1^2\Phi^*_{k_1}T^*_{k_2}\right)\right],$$

where T_k, Φ_k are Fourier components of ion temperature and electrostatic potential perturbations which are normalized as follows

$$\hat{T} = \frac{L_n}{\rho_s}\frac{\delta T}{T}, \quad \hat{\Phi} = \frac{L_n}{\rho_s}\frac{e\Phi}{T}, \quad T = T_e = T_i.$$

Dimensionless time and space coordinate are $t \to \omega_s\cdot t$, $\bar{r}\to\bar{r}/\rho_s$, $\omega_s = c_s/L_n$, $\rho_s = c_s/\Omega_{ci}$. Here we have used notations

$$A = \frac{\varepsilon_n}{3}(7 - 2k^2),$$
$$B = \frac{4}{3}\varepsilon_n(1-k^2) - \eta_i + \frac{2}{3}(2+\eta_i)k^2,$$
$$C = \varepsilon_n(1-k^2),$$
$$D = 2C - \frac{1}{1+k^2}\left(1 - (1+\eta_i)k^2\right),$$

$$\varepsilon_n = \frac{2d \ln B}{d \ln n}.$$

In the linear approximation one can put Φ_k, $T_k \sim \exp[-i\omega_k t]$ and obtain the linear dispersion reaction for η_i modes

$$\omega_k^{\pm} = \frac{1}{2} k_y (A + D) \pm \frac{1}{2} k_y \sqrt{(A - D)^2 + 4BC} =$$

$$= \frac{1}{2} k_y \left(1 - \frac{13}{3}\varepsilon_n\right) - k^2 \frac{3}{8}\left(\varepsilon_n + \eta_i\right) \pm \frac{1}{2} k_y \delta_k, \quad (2)$$

where $\delta_k = 2\sqrt{\varepsilon_n(\eta_{icr} - \eta_i)}$, $\eta_{icr} = \eta^0_{icr} + k^2 \times \left[2 - \frac{5}{18}\varepsilon_n - \frac{\eta_i}{2}\left(1 + \frac{1}{\varepsilon_n}\right)\right]$, $\eta^0_{icr} = \frac{1}{6} + \frac{49}{36}\varepsilon_n + \frac{1}{4\varepsilon_n}$.
The curve $\eta^0_{icr}(\varepsilon_n)$ gives the marginal stability boundary for η_i mode. If $\eta < \eta^0_{icr}$ these modes are linearly stable.

For weak nonlinearities one can obtain from (1) the set of equations for amplitudes Φ_k, Φ_{k_1}, Φ_{k_2} of three interacting waves satisfying resonant conditions $\omega_k + \omega_{k1} + \omega_{k2} = 0$, $\mathbf{k} + \mathbf{k}_1 + \mathbf{k}_2 = 0$ in the region of linear stability:

$$\hat{L}\Phi_k = V_{kk_1k_2} \Phi^*_{k_1} \Phi^*_{k_2},$$

$$\hat{L}\Phi_{k_1} = V_{k_1k_2k} \Phi^*_{k} \Phi^*_{k_2},$$

$$\hat{L}\Phi_k = V_{k_2kk_1} \Phi^*_{k} \Phi^*_{k_1}, \quad (3)$$

where

$$\hat{L}\Phi_k = \left\{\pm i k_y \delta_k \left(\frac{\partial}{\partial t} - \frac{\omega_k}{k_y}\frac{\partial}{\partial y}\right) - \left(\frac{\partial}{\partial t} - \frac{\omega_k}{k_y}\frac{\partial}{\partial y}\right)^2 - \right.$$

$$\left. - k_y^2 a_k \left(2i\left(k_x \frac{\partial}{\partial x} + k_y \frac{\partial}{\partial y}\right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\right\}\Phi_k,$$

$$a_k = -\frac{\omega_k}{k_y}\left(\eta_i + \frac{16}{3}\varepsilon_n\right) + 10\varepsilon_n + 8\eta_i - 2,$$

$$V_{kk_1k_2} = \left(\mathbf{k}_1 \times \mathbf{k}_2\right)\cdot \mathbf{e}_{\parallel}\left(k_2^2 - k_1^2\right) V, \quad V = 2\varepsilon_n - \frac{2}{3} - \frac{\eta_i}{4}.$$

The signs $\pm$ in the first term of operator $\hat{L}$ corresponds to modes ω_k^+ or ω_k^-.

Sufficiently far from marginal stability boundary or for sufficiently small wave amplitudes:

$$t_0 \varepsilon_n^{1/2}\left(\eta^0_{icr} - \eta_i\right) \gg 1 \text{ or } t_0\left(\omega_k^+ - \omega_k^-\right) \gg 1, \quad (4)$$

the temporal development of instability was considered before [2].

In other limiting case

$$t_0\left(\omega_k^+ - \omega_k^-\right) \ll 1 \quad (5)$$

the temporal development of nonlinear instability is described by

$$\frac{d^2\Phi_k}{dt^2} = -V_{kk_1k_2}\Phi^*_{k_1}\Phi^*_{k_2}. \quad (6)$$

For equal signs of matrix elements: $V_{kk_1k_2}$, $V_{k_1k_2k}$, $V_{k_2kk_1}$ the system (6) has simple solution describing modified explosive instability which was found in [4,5]:

$$\Phi_k = \frac{6}{\sqrt{\left|V_{k_1k_2k}V_{k_2kk_1}\right| (t_0 - t)^2}},$$

Φ_{k_1}, Φ_{k_2} have similar form. If one of the matrix element, say $V_{kk_1k_2}$, is of opposite sign, the system (6) after normalization can be written in the form

$$\frac{d^2C}{dt^2} = C_1C_2, \quad \frac{d^2C_1}{dt^2} = -C^*C_2^*, \quad \frac{d^2C_2}{dt^2} = -C^*C_1^*. \quad (7)$$

It also has exact solution, describing "explosion" [6] (see Fig.1)

$$C = \frac{\sqrt{84}\, e^{i\alpha}}{(t_0 - t)^{2 - i2\sqrt{3}}}, \quad C_1 = C_2 = \frac{\sqrt{168}\, e^{i\beta}}{(t_0 - t)^{2 - i\sqrt{3}}}, \quad (8)$$

where α and β — constants.

For any other initial values of $C_i(0)$, $C'_i(0)$, $\varphi_i(0)$, $\varphi'_i(0)$ solution of (7) tends to self-similar solution (8) asymptotically when $t \to t_0$ (see Fig.2). Characteristic feature of such solution is that phases $|\varphi_i|$ grows with time as well as wave intensities. The system (7) has also solutions with constant phases. They have been found numerically (see Fig.3). The most simple one can be estimated as $C \sim at^{4/3}$, $C_1 \approx C_2 \sim \sqrt{8a/3}\, t^{-1/3} \cos(3\sqrt{a}\, t^{5/3}/5 + \theta_0)$, where a, θ_0 are constants.

Temporal description of instability is valid only if we can consider wave interaction with amplitudes which does not depend on space coordinate. In reality this is not so and we should consider spatial-temporal problem to describe evolution of wave amplitudes. In this case one can obtain obvious generalization of the

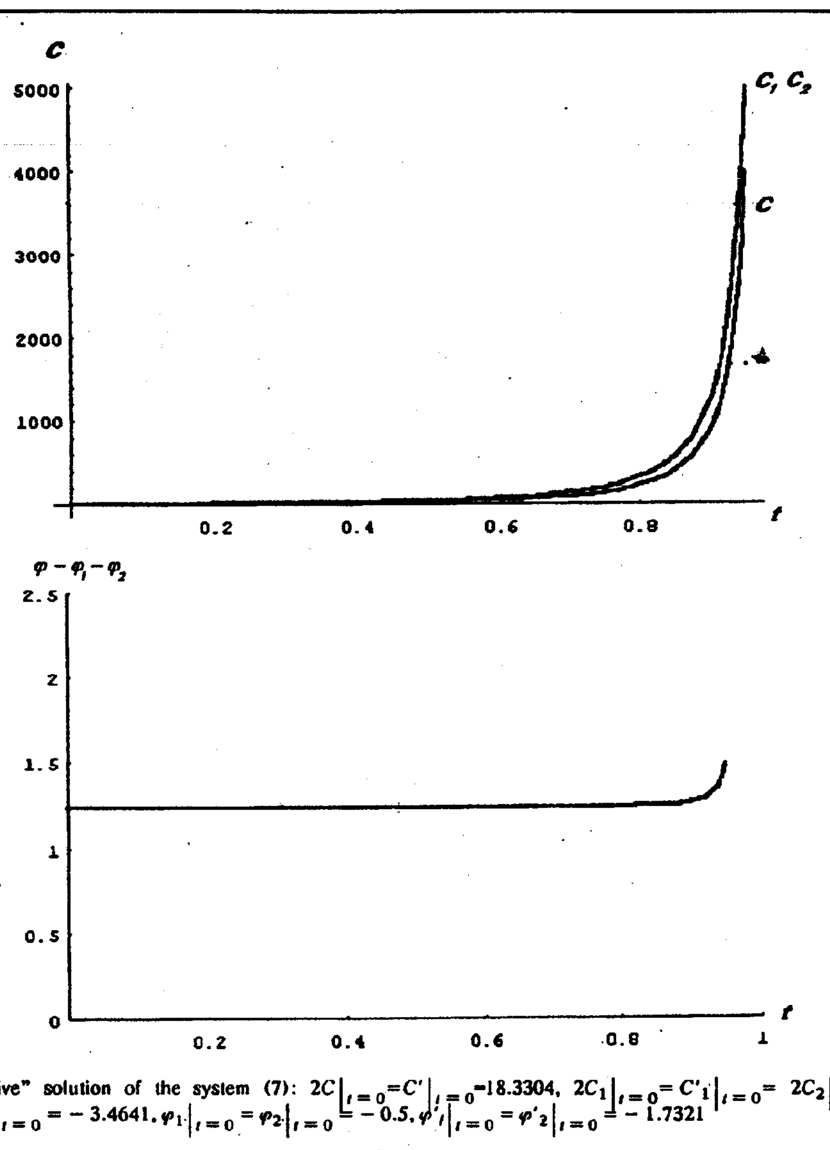

Fig.1. Exact "explosive" solution of the system (7): $2C|_{t=0}=C'|_{t=0}=18.3304$, $2C_1|_{t=0}=C'_1|_{t=0}=2C_2|_{t=0}=C'_2|_{t=0}=25.922$, $\varphi|_{t=0}=0.2373$, $\varphi'|_{t=0}=-3.4641$, $\varphi_1|_{t=0}=\varphi_2|_{t=0}=-0.5$, $\varphi'_1|_{t=0}=\varphi'_2|_{t=0}=-1.7321$

system (6) which follows from (3) under condition (5)

$$-\left(\frac{\partial}{\partial t}-\frac{\omega_k}{k_y}\frac{\partial}{\partial x}\right)^2\Phi_k=V_{kk_1k_2}\Phi^*_{k_1}\Phi^*_{k_2} \tag{9}$$

and similar equations for Φ_{k_1} and Φ_{k_2}.

As shown in [8] for given boundary conditions at the $x = 0$ of the interval $(0, L)$ steady state is established through time $L / \min |\omega_k/k_y|$ if L is less than the "explosive length" for the corresponding "spatial" problem of the system (9) and if the signs of matrix elements in (9) and signs of wave velocities ω_k/k_y are the same. Now we have checked that it is true for any signs of matrix elements $V_{kk_1k_2}$ and for equal signs of ω_k/k_y (see Fig.4).

We have shown also that system (3) may describe solitons moving with constant velocity U in the direction of drift wave propagation $(\hat{y})$. To find it we choose $\Phi_k(x, y, t)$ in the form

$$\Phi_{k_i}=\Psi_{k_i}\exp\left(i\left(s_{k_i}y+q_{k_i}x\right)\right),$$

where $\eta = y + Ut$; U, s_{k_i}, q_{k_i} are constants which will be determined below. First of all we require

$$s_k+s_{k_1}+s_{k_2}=0,\quad q_k+q_{k_1}+q_{k_2}=0. \tag{10}$$

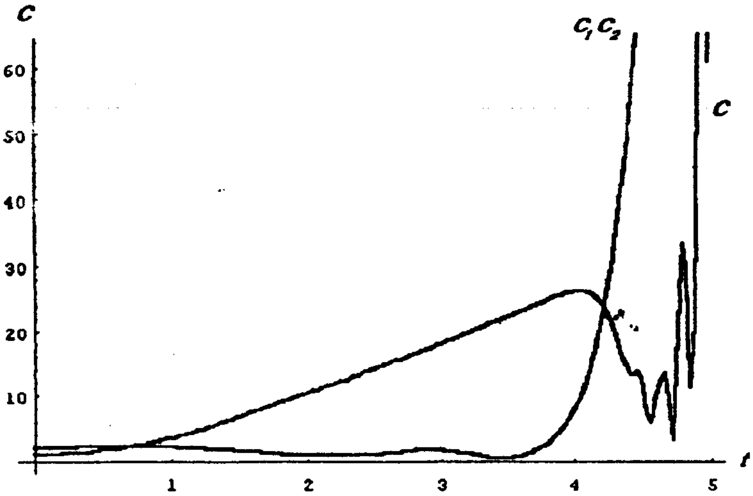

Fig.2. Asymptotic "explosive" solution of the system (7): $C|_{t=0} = C'_1|_{t=0} = C'_2|_{t=0} = 0$, $C|_{t=0} = 1$, $C_1|_{t=0} = C_2|_{t=0} = 2$, $\varphi_i|_{t=0} = \varphi'_i|_{t=0} = 0$

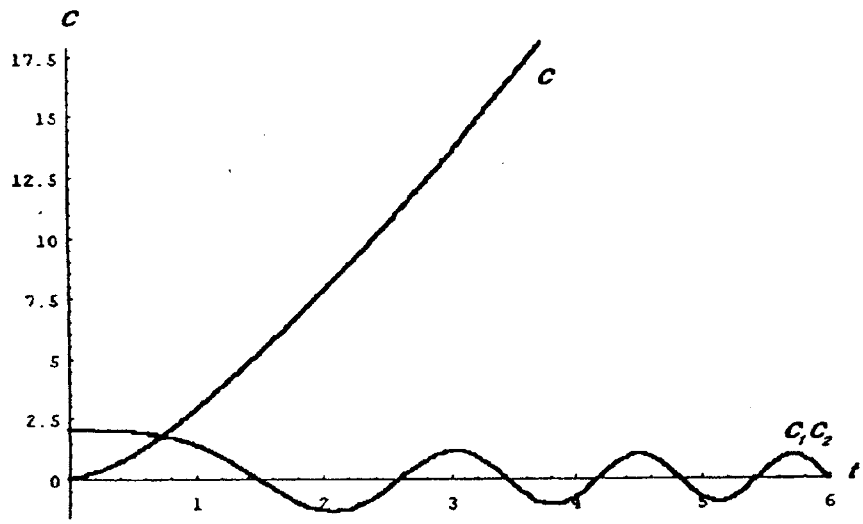

Fig.3. "Nonexplosive" solution of the system (7): $C|_{t=0} = C'|_{t=0} = C'_1|_{t=0} = C'_2|_{t=0} = 1$, $C_i|_{t=0} = C_2|_{t=0} = 2$, $C'|_{t=0} = 1$, $\varphi_i|_{t=0} = 1$, $\varphi'_i|_{t=0} = 0$

From (3) we obtain following set of equations for Ψ_{k_i}:

$$\left[1 + \frac{(U-u_k)^2}{a_k}\right]\frac{d^2\Psi_k}{d\eta^2} - 2i\left[\frac{u_k(U-u_k)\,s_k}{a_k} + \frac{k_y\delta_k(U-u_k)}{2a_k} + k_y - s_k\right]\frac{d\Psi_k}{d\eta} -$$

$$- \left[\frac{u_k^2 s_k^2}{a_k} + \frac{k_y\delta_k u_k s_k}{a_k} - 2k_x q_k + q_k^2 - 2k_y s_k + s_k^2\right]\Psi_k =$$

$$= \frac{V_{kk_1k_2}}{a_k}\Psi^*_{k_1}\Psi^*_{k_2} \qquad (11)$$

and similar equations for Ψ_{k_1}, Ψ_{k_2}.

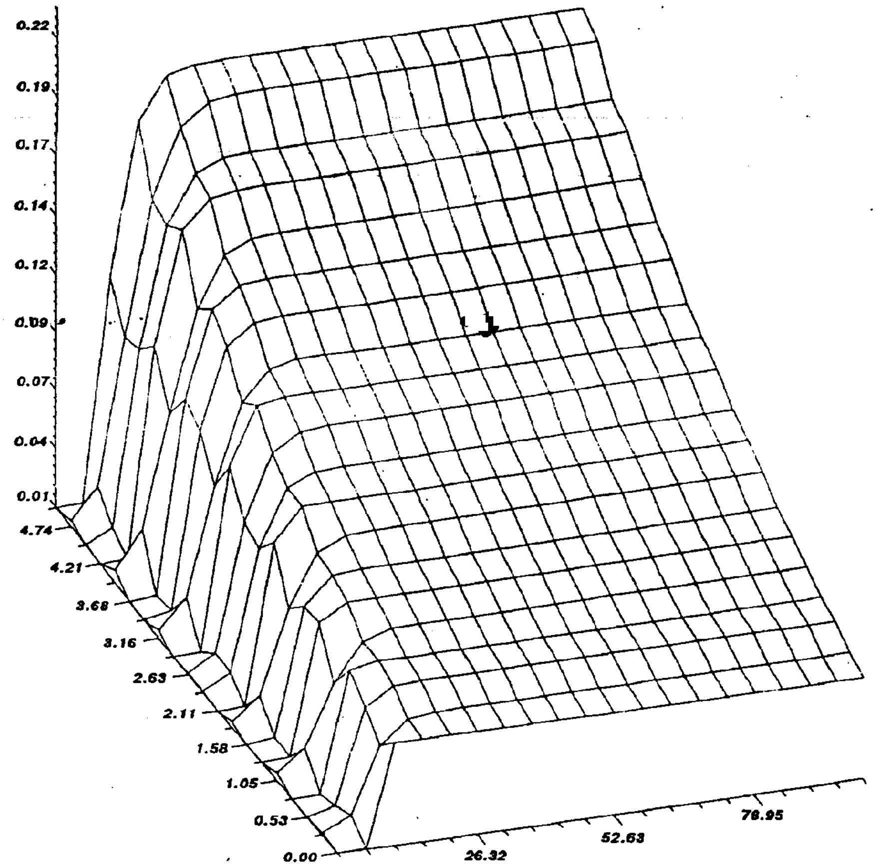

Fig. 4. Saturation of explosive instability: $\Phi_i|_{t=0,x} = 0.015$, $\partial \Phi_i/\partial t|_{t=0,x} = 0.0015$, $\Phi_i|_{t,x=0} = 0.06$, $\partial \Phi_i/\partial x|_{t,x=0} = 0.012$, $\omega_k/k_y = 0.95$, $\omega_{k_1}/k_{1y} = 1$, $\omega_{k_2}/k_{2y} = 0.9$

Then we require that coefficient before $\frac{d\Psi_k}{d\eta}$ vanish

$$s_k\left[1 - \frac{u_k(U - u_k)}{a_k}\right] = k_y\left[1 + \frac{1}{2}\,\frac{\delta_k}{a_k}(U - u_k)\right]$$

and similar equations for s_{k_1}, s_{k_2}.

The first condition (10) gives cubic equation to determine U. To simplify it we assume that $(U - u_k)\,u_k/a_k \ll 1$, $(U - u_k)^2/a_k \ll 1$, $\delta_k \ll u_k$ which are very easy to fulfill near stability boundary.

Then we can obtain

$$U = \frac{k_y\frac{u_k^2}{a_k} + k_{1y}\frac{u_{k_1}^2}{a_{k_1}} + k_{2y}\frac{u_{k_2}^2}{a_{k_2}}}{k_y\frac{u_k}{a_k} + k_{1y}\frac{u_{k_1}}{a_{k_1}} + k_{2y}\frac{u_{k_1}}{a_{k_2}}}. \tag{12}$$

We intend to reduce the system (11) to the more symmetric one:

$$\frac{d^2\Psi_k}{d\eta^2} - \lambda\,\Psi_k = \frac{V_{kk_1k_2}}{a_k}\Psi^*_{k_1}\Psi^*_{k_2},$$

$$\frac{d^2\Psi_{k_1}}{d\eta^2} - \lambda\,\Psi_{k_1} = \frac{V_{k_1k_2k}}{a_{k_1}}\Psi^*_k\Psi^*_{k_2},$$

$$\frac{d^2\Psi_{k_2}}{d\eta^2} - \lambda\,\Psi_{k_2} = \frac{V_{k_2kk_1}}{a_{k_2}}\Psi^*_k\Psi^*_{k_1}. \tag{13}$$

Comparing (11) and (13) one can determine

$$2k_{xi}q_{k_i} + k_{iy}^2 - \frac{u_{k_i}^2k_{iy}^2}{a_{k_i}} = -\lambda.$$

Taking into account the second equation (10) we obtain

$$\lambda = \frac{1}{3}\left\{\frac{u_k^2 k_y^2}{a_k} + \frac{u_{k_1}^2 k_{1y}^2}{a_{k_1}} + \frac{u_{k_2}^2 k_{2y}^2}{a_{k_2}} + k_y^2 - k_{1y}^2 - k_{2y}^2\right\}. \quad (14)$$

So we are able to determine all unknown values U, s_{k_i}, q_{k_i}.

The system (13) can be written in the most symmetric form for functions $\varkappa_k = -\left(\frac{V_{k_2kk_1}V_{k_1k_2k}}{a_{k_2}a_{k_1}}\right)^{1/2} \Psi_k \operatorname{sign}\left(\frac{V_{kk_1k_2}}{a_k}\right)$ if all signs of $\frac{V_{kk_1k_2}}{a_k}$, $\frac{V_{k_1k_2k}}{a_{k_1}}$, $\frac{V_{k_2kk_1}}{a_{k_2}}$ are equal. We have for $\varkappa_k$:

$$\frac{d^2\varkappa_k}{d\eta^2} - \lambda\varkappa_k + \varkappa_{k_1}^*\varkappa_{k_2}^* = 0 \quad (15)$$

and similar equations for $\varkappa_{k_1}$, $\varkappa_{k_2}$. The soliton solution of KdV equation (15) is well-known:

$$\varkappa_k = \varkappa_{k_1} = \varkappa_{k_2} = \frac{3\lambda}{2\cosh^2\left[\frac{\lambda^{1/2}}{2}(y + Ut - \eta_0)\right]},$$

when U and λ is determined by (12) and (14), η_0 is a constant. So for the drift wave amplitude Ψ_k we have:

$$\Psi_k = -\operatorname{sign}\left(\frac{V_{kk_1k_2}}{a_k}\right)\left(\frac{a_{k_1}a_{k_2}}{V_{k_1k_2k}V_{k_2k_1k}}\right)^{1/2} \times$$

$$\times \frac{3\lambda}{2\cosh^2\left[\frac{\lambda^{1/2}}{2}(y + Ut - \eta_0)\right]},$$

and similar expressions for Ψ_{k_1}, Ψ_{k_2}.

Thus we have three bounded envelope solitons moving with the same velocity U (12) which is of order of phase velocity of drift waves. One can also find periodic nonlinear solutions of the system (15). We see that modified explosive instability can be stabilized by forming spatial nonlinear structures which would constitute structural elements of subcritical turbulence.

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ОСОБЛИВОСТІ ВЗАЄМОДІЇ НЕЛІНІЙНИХ ДРЕЙФОВИХ ХВИЛЬ У ПЛАЗМІ ПОБЛИЗУ ГРАНИЦІ ЛІНІЙНОЇ НЕСТІЙКОСТІ

Т. О. Давидова, О. Ю. Панькін

Р е з ю м е

Показано, що динаміка нелінійної взаємодії дрейфових хвиль суттєво змінюється, коли енергія взаємодії перевищує власну енергію хвиль. Знайдено умови насичення модифікованої вибухової нестійкості внаслідок конвективного виносу хвиль з області взаємодії. Доведена можливість утворення солітонів та інших стаціонарних хвильових структур під час нелінійної взаємодії поблизу границі лінійної нестійкості плазми.

ОСОБЕННОСТИ ВЗАИМОДЕЙСТВИЯ НЕЛИНЕЙНЫХ ДРЕЙФОВЫХ ВОЛН В ПЛАЗМЕ ВБЛИЗИ ГРАНИЦЫ ЛИНЕЙНОЙ НЕУСТОЙЧИВОСТИ

Т. А. Давыдова, А. Ю. Панькин

Р е з ю м е

Показано, что динамика нелинейного взаимодействия дрейфовых волн существенно изменяется, когда энергия взаимодействия превышает собственную энергию волн. Найдены условия насыщения модифицированной взрывной неустойчивости вследствие конвективного выноса волн из области взаимодействия. Показана возможность образования солитонов и других стационарных волновых структур при нелинейном взаимодействии вблизи границы линейной неустойчивости плазмы.

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