ALPHA PARTICLE DRIVEN TAE INSTABILITY SUPPRESSION BY A RESONANT RF FIELD

V.S. MARCHENKO, A.Yu. PAN'KIN
Institute for Nuclear Research,
Academy of Sciences of Ukraine,
Kiev, Ukraine

ABSTRACT. The toroidicity induced shear Alfvén eigenmode (TAE) instability driven by the fusion alpha particle pressure gradient is found to be stabilized in the presence of a resonant RF field with $\Omega = k_{0\perp} \cdot v_{\alpha\perp}$, where Ω is the frequency, k_0 the wavenumber and $v_{\alpha\perp}$ is the α particle velocity component perpendicular to the magnetic field. This kind of resonance could occur during lower hybrid (LH) current drive in an ignited tokamak plasma. The RF electric field strength that is needed for stabilization is available from modern RF power sources.

1. INTRODUCTION

One of the major issues in the alpha particle physics of tokamaks is the low n toroidicity induced Alfvén eigenmode (TAE) instability driven by the α particle pressure gradient and the resultant α particle transport [1–5]. The main results of the previous studies of this problem are that the volume averaged α particle β threshold for TAE instability is small, on the order of 10^{-4}, and that for a relatively low fluctuation level with $\delta B_r/B_0 \geq 10^{-4}$ the α particle loss time is comparable with, and even shorter than, the α particle slowing down time. This is the reason that the problem of efficient suppression of this instability seems to be important.

Several damping mechanisms of TAE modes have been considered in the literature. They are: electron Landau damping [1], ion Landau damping [4], continuum damping [6] and collisional damping on trapped electrons [7]. The last three mechanisms taken together could increase the α particle β threshold by more than an order of magnitude. In the present work we consider an alternative damping mechanism that should arise in the stationary tokamak reactor when the plasma current is driven by lower hybrid (LH) waves, which is the most promising current drive scheme at present. It is well known [8] that α particles could interact with LH modes via the transverse Landau resonance, $v_{ph\perp} = v_{\alpha\perp}$, where $v_{ph\perp}$ and $v_{\alpha\perp}$ are perpendicular to the equilibrium magnetic field velocity components of the LH wave phase and the α particle, respectively. Such interactions have been observed recently during LHCD experiments in JET [9]. The interaction results in quasi-linear diffusion in the resonant part of the α particle phase space. The quasi-linear diffusion operator modifies the α particle response on the Alfvén perturbation in such a way that the TAE mode becomes heavily damped for moderate values of LH power. This damping mechanism has been reported previously [10], when the continuum Alfvén waves instability on the trapped α particles was considered. Here we are extending the theory to the ($m = -2$, $n = -1$) TAE instability driven by resonance with passing α particles.

In the following section, the drift kinetic equation for the non-adiabatic perturbed α particle distribution is solved in the 'banana' regime ($\nu_{QL} < \epsilon\omega_A < \omega_{b(t)\alpha}$, where ν_{QL} is the characteristic frequency of α particle scattering due to quasi-linear processes, $\epsilon = a/R$ is the inverse aspect ratio, ω_A is the Alfvén frequency and $\omega_{b(t)\alpha}$ is the α bounce (transit) frequency. The correct α particle response driven by the toroidicity induced shear Alfvén eigenmode is derived in Section 3. The TAE stability analysis that uses perturbation theory is carried out in Section 4. Conclusions are given in Section 5.

2. SOLUTION OF THE DRIFT KINETIC EQUATION

As was mentioned in the Introduction, the quasi-linear diffusion effect on the trapped α particle perturbative distribution in the presence of continuum shear Alfvén wave has been considered in Ref. [10]. Here we briefly repeat that calculation for the passing α particle response.

The drift kinetic equation for the non-adiabatic perturbed α particle distribution can be written in the form

$$(\omega - \langle\omega_{\rm d}\rangle - {\rm i}\langle C_{\rm QL}\rangle)f_\alpha^{\rm t} = -\frac{\omega}{2R}\exp[{\rm i}(m-S)\theta - n\varphi]\left(\frac{\partial F_\alpha}{\partial\epsilon} + \frac{m}{r\omega\omega_{{\rm B}\alpha}}\frac{\partial F_\alpha}{\partial r}\right)$$

$$\times\left\langle(\epsilon_\perp + v_\parallel^2)\left(\xi_m\cos\theta - {\rm i}\frac{r}{m}\frac{{\rm d}\xi_m}{{\rm d}r}\sin\theta\right){\rm e}^{{\rm i}s\cdot\theta}\right\rangle \quad (1)$$

$$\left[\omega - \langle\omega_{\rm d}\rangle \mp \left(k_\parallel + \frac{S}{qR}\right)\langle|v_\parallel|\rangle - {\rm i}\langle C_{\rm QL}\rangle\right]f_{\alpha\pm}^{\rm u} = -\frac{\omega}{2R}\exp[{\rm i}(m-S)\theta - n\varphi]$$

$$\times\left(\frac{\partial F_\alpha}{\partial\epsilon} + \frac{m}{r\omega\omega_{{\rm B}\alpha}}\frac{\partial F_\alpha}{\partial r}\right)\left\langle(\epsilon_\perp + v_\parallel^2)\left(\xi_m\cos\theta - {\rm i}\frac{r}{m}\frac{{\rm d}\xi_m}{{\rm d}r}\sin\theta\right){\rm e}^{{\rm i}S\theta}\right\rangle \quad (2)$$

where $f_\alpha^{\rm t(u)}$ is the perturbed distribution of trapped (untrapped) α particles with $f_{\alpha\pm}^{\rm u}$ corresponding to $v_\parallel \gtrless 0$, ξ_m is the mth harmonic of the radial component of plasma displacement $\xi = \exp(im\theta - in\varphi)\,\xi_m$, m, n are integers, $S = \pm1$, θ, φ are the generalized poloidal and toroidal angles, respectively, $\omega_{\rm d}$ is the magnetic drift frequency, $C_{\rm QL}$ is the quasi-linear diffusion operator, F_α is the α particle equilibrium distribution, $\epsilon_\perp = v_\perp^2/2$ and $\langle A\rangle$ denotes the bounce or transit average of A and is given by

$$\langle A\rangle = \frac{1}{\tau_{\rm b(t)}}\oint\frac{A\,{\rm d}l}{|v_\parallel|}$$

where $\tau_{\rm b(t)}$ is the bounce (transit) period of trapped (circulating) α particles.

Equations (1) and (2) represent ordinary differential equations in velocity space for three α particle species which should be solved and matched across the separatrix. For typical parameters of tokamak plasma and external RF field the 'banana' regime ($\nu_{\rm QL} < (r/R)\,\omega_{\rm A}$) is realized. In this regime a narrow ($|\Delta v_\parallel/v_\parallel| < (\nu_{\rm QL}R/r\omega_{\rm A})^{1/2}$) boundary layer forms at the trapped–untrapped boundary in velocity space, so that only α particles from this layer are significantly scattered by quasi-linear processes. This problem is similar to that considered in Refs [11, 12], where the trapped electron modifications to tearing [11] and Alfvén [12] modes due to Coulomb collisions were calculated. Hereafter we follow the same procedure in the calculation of the α particle response to the TAE perturbation in the presence of a resonant RF field.

The asymptotic expression for the bounce or transit averaged quasi-linear operator near the separatrix is given by Ref. [10] as

$$\langle C_{\rm QL}\{f_\alpha\}\rangle_{\rm t(u)} = 2\epsilon^{1/2}\left(\frac{e_\alpha}{m_\alpha}\right)^2|E_-|^2\left(\frac{\omega_0}{k_0}\right)^2\frac{1}{\tau_{\rm b(t)}B_0^2(1+\epsilon)}\frac{\partial}{\partial\mu}\left(\frac{\tau_{\rm b(t)}}{[2k_0^2\mu B_0(1+\epsilon) - \Omega^2]^{1/2}}\frac{\partial f_\alpha}{\partial\mu}\right) \quad (3)$$

where $\epsilon = r/R$, and E_-, Ω and k_0 are the RF electric field amplitude, frequency and wavenumber, respectively, while $\mu = v_\perp^2/2B$ and $B = B_0(1 - \epsilon\cos\theta)$. The general solution of Eqs (1) and (2) could be presented as the sum of a particular solution of the non-homogeneous equation and the general solution of the homogeneous one. In the 'banana' regime the zero order ($\nu_{\rm QL} = 0$) non-homogeneous solution is a good approximation, while the homogeneous part represents the contribution of quasi-linear diffusion to the response. Assuming that the resonances are possible only with passing α particles (i.e. neglecting the $\langle\omega_{\rm d}\rangle$ terms in Eqs (1) and (2)) one could obtain the homogeneous equations in the form

$$\left(\omega - {\rm i}\nu_{\rm QL}\frac{1}{K(\kappa)}\frac{\partial}{\partial\kappa^2}K(\kappa)\frac{\partial}{\partial\kappa^2}\right)f_\alpha^{\rm t} = 0, \qquad \kappa^2 < 1 \quad (4)$$

$$\left[\omega \mp \left(k_\parallel + \frac{S}{qR}\right)\langle|v_\parallel|\rangle - {\rm i}\nu_{\rm QL}\frac{1}{K(\kappa^{-1})}\frac{\partial}{\partial\kappa^2}K(\kappa^{-1})\frac{\partial}{\partial\kappa^2}\right]f_\alpha^{\rm u} = 0, \qquad \kappa^2 > 1 \quad (5)$$

where $\nu_{\rm QL} = 2\epsilon^{1/2}(e_\alpha/m_\alpha)^2|E_-|^2(\Omega/k_0)^2(\epsilon v^2)^{-2}(k_0^2v^2 - \omega_0^2)^{-1/2}$, $\kappa^2 = [\varepsilon - \mu B_0(1+\epsilon)]/2\epsilon\mu B_0$, $\varepsilon = v^2/2$ and $K(\kappa)$ is the full elliptic integral of the first kind.

With the change of variables

$$\tau^2 = 256{\rm i}\frac{\omega}{\nu_{\rm QL}}y^2\begin{cases}1, & \kappa^2 < 1\\ 1 \pm \dfrac{4\pi(k_\parallel + S/qR)}{\ln y}\dfrac{(\epsilon\varepsilon)^{1/2}}{\omega}, & \kappa^2 > 1\end{cases}$$

where $y_t = (1 - \kappa^2)/16$ and $y_u = (1 - \kappa^{-2})/16$. Equations (4) and (5) become

$$\frac{d^2 f_\alpha}{d\tau^2} + \frac{1}{\tau \ln y(\tau)} \frac{d f_\alpha}{d\tau} + f_\alpha = 0 \tag{6}$$

Typically, $|\ln y| \approx \frac{1}{2}|\ln(\nu_{QL}/256\omega)| \gg 1$, so that an approximate solution to Eq. (6) that decays to zero away from the separatrix is $f_\alpha = \exp(i\tau)$. As a result, the lowest order solution to Eqs (1) and (2) is

$$f_\alpha = f_\alpha^{(0)} + A e^{i\cdot\tau} \tag{7}$$

where $f_\alpha^{(0)}$ is the particular solution (for $\nu_{QL} = 0$) and the constant A represents the three constants A_t and $A_\pm$ to be determined by matching.

In this work we restrict our consideration by the condition $\omega_d < \omega < \omega_{b\alpha}$, so that both magnetic drift and bounce harmonic resonances are excluded. The second inequality means that the trapped α particle distribution is an approximately even function (the odd part of the trapped α particle response is higher order in the bounce harmonic expansion). Thus, to determine the three constants A_t and $A_\pm$, both the even and the odd parts of the distribution function f_α plus the derivative of the even part of f_α must be matched across the trapped/untrapped boundary at $\tau = 0$. This matching procedure is similar to that carried out in Ref. [11] for the trapped electron modifications to tearing modes in the limit of weak Coulomb collisions. The expression for $A_\pm$ obtained by matching is given by

$$(2 + \Delta_+ + \Delta_-)A_\pm = 2(f_{\alpha t}^{(0)} - f_{\alpha\pm}^{(0)}) + (f_{\alpha\mp}^{(0)} - f_{\alpha\pm}^{(0)})\Delta_\mp \tag{8}$$

where

$$\Delta_\pm = \left[1 \mp 4\pi\left(k_\parallel + \frac{S}{qR}\right)\ln^{-1}\left(\frac{256\omega}{\nu_{QL}}\right)\frac{(\epsilon\varepsilon)^{1/2}}{\omega}\right]^{1/2}$$

In the next section using solution (7), (8) we derive the α particle response driven by the toroidicity induced shear Alfvén eigenmode in the presence of a resonant RF field.

3. ALPHA PARTICLE RESPONSE

In order to incorporate the contribution from α particle quasi-linear diffusion to the basic eigenmode equations, we follow the procedure described in Ref. [13], where the α particle kinetic response to the global Alfvén eigenmode (GAE) was calculated in terms of the susceptibility tensor $\hat{\chi}_\alpha$

$$\sigma_\alpha \equiv \int d^3v\, \boldsymbol{v}_{d\alpha} \nabla f_\alpha = \nabla_\perp \boldsymbol{j}_\alpha = \nabla_\perp[(\omega/4\pi i)\,\hat{\chi}_\alpha \boldsymbol{E}] \tag{9}$$

with $\boldsymbol{E}$ as the mode electric field. Substituting (7) and (8) into (9) one can obtain for the mth harmonic of σ_α^{QL},

$$\frac{4\pi i}{\omega}\sigma_\alpha^{QL} = \frac{\partial}{\partial r}(-L_{m-1}E_r + iS_{m-1}E_\theta) - \frac{m-1}{r}(-S_{m-1}E_r + iK_{m-1}E_\theta)$$

$$+ \frac{\partial}{\partial r}(-L_{m+1}E_r - iS_{m+1}E_\theta) - \frac{m+1}{r}(S_{m+1}E_r + iK_{m+1}E_\theta) \tag{10}$$

where

$$\begin{Bmatrix} L_{m\pm1} \\ S_{m\pm1} \\ K_{m\pm1} \end{Bmatrix} = \begin{Bmatrix} \langle\sin^2\theta\rangle^2 \\ \langle\sin^2\theta\rangle\langle\cos^2\theta\rangle \\ \langle\cos^2\theta\rangle^2 \end{Bmatrix} \frac{\beta_\alpha}{2R^2}\frac{c^2}{\omega^2}\frac{1}{v_\alpha^4}\int d^3v \frac{(v_\perp^2/2 + v_\parallel^2)^2}{2 + \Delta_+ + \Delta_-}\left(-T_\alpha\frac{\partial \bar{F}_\alpha}{\partial\varepsilon} - \frac{\omega_{*\alpha m}}{\omega}\bar{F}_\alpha\right)$$

$$\times\left\{\left[2\left(1 - \frac{1}{\Delta_+^2}\right) + \left(\frac{1}{\Delta_-^2} - \frac{1}{\Delta_+^2}\right)\Delta_-\right] H(v_\parallel)\exp\left[i(1+i)\left(\frac{\omega}{2\nu_{QL}}\right)^{1/2}\Delta_+(1 - \kappa^{-2})\right]\right.$$

$$\left. + \left[2\left(1 - \frac{1}{\Delta_-^2}\right) + \left(\frac{1}{\Delta_+^2} - \frac{1}{\Delta_-^2}\right)\Delta_+\right] H(-v_\parallel)\exp\left[i(1+i)\left(\frac{\omega}{2\nu_{QL}}\right)^{1/2}\Delta_-(1 - \kappa^{-2})\right]\right\} \tag{11}$$

where

$$\beta_\alpha = 8\pi n_\alpha T_\alpha / B^2, \quad \int d^3 v \bar{F}_\alpha = 1, \quad v_\alpha^2 = 2T_\alpha/m_\alpha, \quad \omega_{*\alpha m} = \frac{m}{r}\left(\frac{T_\alpha c}{e_\alpha B}\right)\frac{d \ln n_\alpha}{dr}$$

$H(v_\parallel)$ is the unit step function and integration is carried out over the passing particles. Performing the integration over pitch angle, Eq. (11) reduces to

$$\begin{Bmatrix} L_{m\pm1} \\ S_{m\pm1} \\ K_{m\pm1} \end{Bmatrix} = -i \frac{\beta_\alpha}{2R^2}\frac{c^2}{\omega^2}\frac{1}{v_\alpha^4} \begin{Bmatrix} \frac{64}{9}\left[\ln\left(\frac{256\omega}{\nu_{QL}}\right)\right]^{-3/2} \\ \frac{8}{3}\left[\ln\left(\frac{256\omega}{\nu_{QL}}\right)\right]^{-1/2} \\ \left[\ln\left(\frac{256\omega}{\nu_{QL}}\right)\right]^{1/2} \end{Bmatrix} \int_{\frac{1}{2}\left(\frac{\Omega}{k_{0\perp}}\right)^2}^{\frac{1}{4}\frac{R}{r}\left[v_{\alpha 0}^2 - \left(\frac{\Omega}{k_{0\perp}}\right)^2\right]} d\varepsilon\, \varepsilon^{5/2} \left(\frac{\nu_{QL}(\varepsilon)}{\omega}\right)^{1/2} \left(-T_\alpha \frac{\partial \bar{F}_\alpha}{\partial \varepsilon} - \frac{\omega_{*\alpha m}}{\omega}\bar{F}_\alpha\right)$$

$$\times \frac{1}{2 + \Delta_+ + \Delta_-}\left[\frac{2}{\Delta_+}\left(1 - \frac{1}{\Delta_+^2}\right) + \left(\frac{1}{\Delta_-^2} - \frac{1}{\Delta_+^2}\right)\frac{\Delta_-}{\Delta_+} + \frac{2}{\Delta_-}\left(1 - \frac{1}{\Delta_-^2}\right) + \left(\frac{1}{\Delta_+^2} - \frac{1}{\Delta_-^2}\right)\frac{\Delta_+}{\Delta_-}\right] \qquad (12)$$

where we use approximation $v_\perp^2/2 + v_\parallel^2 \approx v_\perp^2/2 \approx \varepsilon$ because the main contribution to the pitch angle integral comes from the thin boundary layer around $\kappa^2 = 1$. The integration limits in (12) are defined by the intersection of the trapped–untrapped boundary $v_\parallel = v_\perp (2r/R)^{1/2}$ with the boundary of the quasi-linear diffusion region in velocity space (region 3 in Fig. 1). It should be noted that the expression between the final pair of square brackets of (12) is negative definite provided $\Delta_\pm^2 > 0$, which corresponds to mode stabilization.

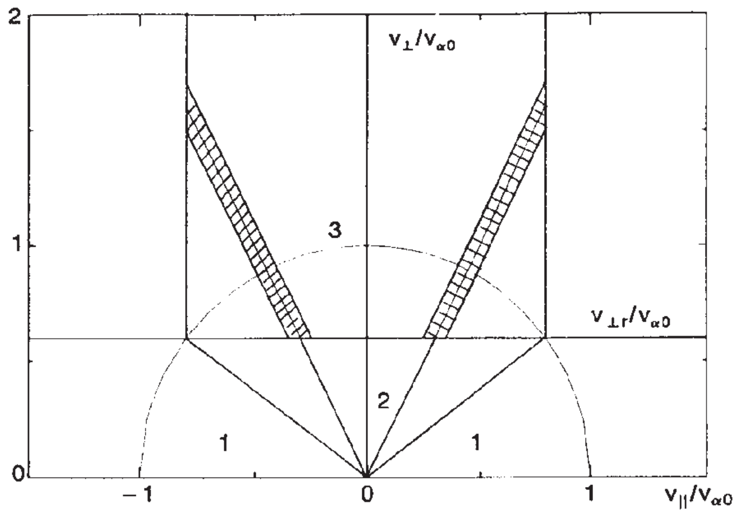

FIG. 1. Velocity space $(v_\perp, v_\parallel)$ divided into three regions. The resonance line $v_\perp = \Omega/k_{0\perp}$ separates the wave-particle interaction region 3 from regions 1 and 2 where the interaction is absent. The hatched regions are boundary layers which are the predominant contributors to TAE damping.

With the help of Ampère's law and the quasi-neutrality condition one can obtain the following two coupled second order eigenmode equations for the poloidal electrical field E_θ, which include kinetic instability drive due to the Cherenkov resonance with passing particles [1] and dissipation due to quasi-linear diffusion in the external RF field (all the other damping mechanisms are dropped):

$$\left\{\frac{d}{dr} r^3 \left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2 - A_{\alpha m} - A_{\alpha m}^{QL}\right)\frac{d}{dr} - r^2 B_{\alpha m}^{QL}\frac{d}{dr} - r\left[(m^2 - 1)\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2 - A_{\alpha m}\right) - C_{\alpha m}^{QL}\right]\right.$$

$$\left. + r^2\left[\left(\frac{\omega^2}{v_A^2}\right)' - A'_{\alpha m} - mB'_{\alpha m} - (A_{\alpha m}^{QL})' - m(D_{\alpha m}^{QL})'\right]\right\} E_m + \epsilon_1 \frac{d}{dr}\left(\frac{\omega^2}{v_A^2}\frac{r^4}{a}\frac{d}{dr}E_{m+1}\right) = 0 \qquad (13)$$

$$\left\{\frac{d}{dr} r^3 \left(\frac{\omega^2}{v_A^2} - k_{\parallel m+1}^2 - A_{\alpha m+1} - A_{\alpha m+1}^{QL}\right) \frac{d}{dr} - r^2 B_{\alpha m+1}^{QL} \frac{d}{dr}\right.$$
$$- r\left[[(m+1)^2 - 1]\left(\frac{\omega^2}{v_A^2} - k_{\parallel m+1}^2 - A_{\alpha m+1}\right) - C_{\alpha m+1}^{QL}\right]$$
$$\left.+ r^2\left[\left(\frac{\omega^2}{v_A^2}\right)' - A'_{\alpha m+1} - (m+1) B'_{\alpha m+1} - (A_{\alpha m+1}^{QL})' - (m+1)(D_{\alpha m+1}^{QL})'\right]\right\} E_{m+1} + \epsilon_1 \frac{d}{dr}\left(\frac{\omega^2}{v_A^2}\frac{r^4}{a}\frac{d}{dr} E_m\right) = 0 \quad (14)$$

where $\epsilon_1 = 3a/2R$, the primes denote radial differentiation, $k_{\parallel m} = (m - nq)/qR$ is the parallel wavenumber with a and R the minor and major radii, respectively, and q the safety factor. The quantities $A_{\alpha m} = Q_{m-1} - Q_{m+1}$ and $B_{\alpha m} = Q_{m-1} + Q_{m+1}$ are derived in Refs [1, 13], where $Q_{m\pm 1}$ is given by

$$Q_{m\pm 1} = -i \frac{\beta_\alpha}{2R^2} \frac{\pi\omega}{v_\alpha^4} \int d^3 v \left(\frac{v_\perp^2}{2} + v_\parallel^2\right)^2 \left(-T_\alpha \frac{\partial \bar{F}_\alpha}{\partial \varepsilon} - \frac{\omega_{*\alpha m}}{\omega} F_\alpha\right) \delta(\omega - k_{\parallel m\pm 1} v_\parallel) \quad (15)$$

and, in addition,

$$A_{\alpha m}^{QL} = (L_{m-1} + L_{m+1}) \frac{\omega^2}{c^2}$$

$$B_{\alpha m}^{QL} = (S_{m-1} + S_{m+1} - L_{m-1} - L_{m+1}) \frac{\omega^2}{c^2}$$

$$C_{\alpha m}^{QL} = [m^2 (K_{m-1} + K_{m+1}) + m (S_{m-1} - S_{m+1} - K_{m-1} + K_{m+1}) - S_{m-1} - S_{m+1}] \frac{\omega^2}{c^2}$$

$$D_{\alpha m}^{QL} = (S_{m-1} - S_{m+1}) \frac{\omega^2}{c^2}$$

where L, S and K are given by (12) with

$$\Delta_{+}^{m\pm 1}(\varepsilon, r) = \left[1 \mp \frac{4\pi [m \pm 1 - nq(r)]}{q(r)R} \ln^{-1}\left(\frac{256\omega}{\nu_{QL}(\varepsilon, r)}\right) \frac{(\varepsilon r/R)^{1/2}}{\omega}\right]^{1/2}$$

$$\nu_{QL}(\varepsilon, r) = \frac{1}{2}\left(\frac{r}{R}\right)^{1/2} \left(\frac{e_\alpha}{m_\alpha}\right)^2 |E_-|^2 \left(\frac{\Omega}{k_{0\perp}}\right)^2 \left(\frac{r}{R}\varepsilon\right)^2 (2k_{0\perp}^2 \varepsilon - \Omega^2)^{-1/2}$$

It should be noted that hereafter we ignore the LH power deposition profile and put $|E_-|^2(r) = \text{const}$. Treating the kinetic terms in (13) and (14) perturbatively, in the next section we analyse the stability of the ($n = -1$, $m = -2$) TAE mode in the presence of an external RF field resonating with α particles.

4. TAE STABILITY ANALYSIS

We begin with the definition of the zero order TAE mode structure. This is an ideal MHD limit (all kinetic terms are dropped). The result is shown in Fig. 2 for $n = -1$, $m = -2$, $\epsilon_1 = 0.375$ with a constant density profile and $q(r) = 1 + (r/a)^2$. It reproduces well the original result of Ref. [1] with $\omega_0 \approx 0.93(|k_{\parallel m} v_A|)|_{q=3/2}$.

Next we consider the kinetic effects of alpha particles on this mode. Exploiting the self-adjointness of the coupled equations, we obtain the imaginary part of the frequency change due to the kinetic terms as follows:

$$\frac{\gamma}{\omega_0} = \frac{v_{A0}^2}{2\omega_0} \frac{\sum_m b_{m1} + b_{m2} + b_{m3}}{\sum_m (d_{m1} + d_{m2}) + d_3} \quad (16)$$

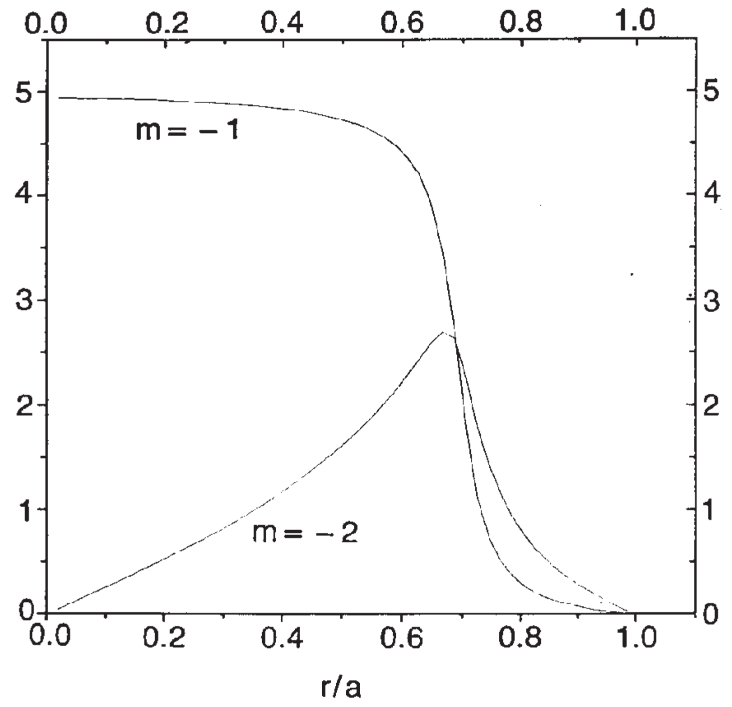

FIG. 2. Radial profiles of the dominant poloidal harmonics for the n = −*1 TAE mode, for a safety factor profile* q = *1* + (r/a)^2 *and a constant density profile.*

where

$$b_{m1} = \int_0^a r^3 E_{0m}'^2 (A_{\alpha m} - A_{\alpha m}^{\rm QL})\,{\rm d}r$$

$$b_{m2} = \int_0^a r^2 E_{0m} E_{0m}' B_{\alpha m}^{\rm QL}\,{\rm d}r$$

$$b_{m3} = \int_0^a \Big([(m^2 - 1) A_{\alpha m} - C_{\alpha m}^{\rm QL}]\, r - r^2 \{A_{\alpha m}' - (A_{\alpha m}^{\rm QL})' + m\,[B_{\alpha m}' - (D_{\alpha m}^{\rm QL})']\} \Big) E_{0m}^2\,{\rm d}r$$

$$d_{m1} = \int_0^a (r^3/\tilde{v}_{\rm A}^2)\, E_{0m}'^2\,{\rm d}r$$

$$d_{m2} = \int_0^a [(m^2 - 1)(r/\tilde{v}_{\rm A}^2) - r^2 (1/\tilde{v}_{\rm A}^2)']\, E_{0m}^2\,{\rm d}r$$

$$d_3 = \int_0^a 2\varepsilon_1 (r^4/a\tilde{v}_{\rm A}^2)\, E_{0m}' E_{0m+1}'\,{\rm d}r$$

and $\tilde{v}_{\rm A}$ is the Alfvén velocity normalized to its value at the centre of the plasma. To calculate the growth rate for the TAE mode using Eq. (16), we need the expression for the equilibrium distribution F_α. The latter satisfies the equation

$$\frac{1}{\tau_{\rm sl} v^2}\frac{\partial}{\partial v}(v^3 + v_{\rm c}^3) F_\alpha + \langle C_{\rm QL}\{F_\alpha\}\rangle + \frac{S_\alpha(r)}{4\pi v_{\alpha 0}^2}\,\delta(v - v_{\alpha 0}) = 0 \qquad (17)$$

where the first term describes α particle slowing down on the background plasma with $\tau_{\rm sl}$ the slowing down time and $v_{\rm c}$ the critical velocity, and the third term represents the α particle source due to DT reactions with $v_{\alpha 0} = (2\varepsilon_{\alpha 0}/m_\alpha)^{1/2}$ and $\varepsilon_{\alpha 0} = 3.52$ MeV. Equation (17) was solved in Ref. [14], the solution could be written in the form (Fig. 1): in region 1 ($|\chi = v_\parallel/v| > \chi_r \equiv (1 - v_{\perp r}^2/v_{\alpha 0}^2)^{1/2}$, $v_{\perp r} = \Omega/k_{0\perp}$)

$$F_\alpha^{(1)}(v) = \frac{S_\alpha \tau_{\rm sl}}{4\pi (v^3 + v_{\rm c}^3)} H(v_{\alpha 0} - v) \qquad (18)$$

in region 2 ($|\chi| < \chi_r$)

$$F_\alpha^{(2)}(v,\chi) = \frac{g v_\perp^3(\chi)}{v^3 + v_c^3} \tag{19}$$

where

$$g = \frac{S_\alpha \tau_{sl}}{4\pi v_\alpha v_{\perp r}^2}, \quad v_r = (v_{\perp r}^2 + v_\parallel^2)^{1/2}$$

in region 3 ($v_\perp > v_{\perp r}$, $C_{QL} \neq 0$)

$$F_\alpha^{(3)}(v_\perp, v_\parallel) = \frac{g}{\mu(v_\perp)}\left(\frac{1}{\Theta_r} + I(v_{\perp r}, v_\parallel) - I(v_{\perp\alpha}, v_\parallel) H(v_\perp - v_{\perp\alpha})\right) \tag{20}$$

where $\Theta(v) = 1 + v_c^3/v^3$, $\Theta_r = \Theta(v_r)$, $v_{\perp\alpha} = (v_{\alpha 0}^2 - v_\parallel^2)^{1/2}$,

$$I(a,b) = \frac{\pi}{2\xi}\frac{v_{\perp r}^2}{v_{\alpha 0}^5}\int_a^b \mu(v_\perp')(v_\perp'^2 - v_{\perp r}^2)^{1/2} v_\perp' \, dv_\perp'$$

$$\mu(v_\perp) = \exp\left(\frac{\pi}{2\xi v_{\alpha 0}^5}\int_{v_{\perp r}}^{v_\perp} \Theta(v')(v_\perp'^2 - v_{\perp r}^2)^{1/2} v_\perp'^3 \, dv_\perp'\right)$$

$$\xi = \frac{\pi}{4}\frac{\tau_{sl} e_\alpha^2 \Omega^2 |E_-|^2}{v_{\alpha 0}^5 m_\alpha^2 k_{0\perp}^3} \equiv \nu_{QL}^{(0)} \tau_{sl}$$

Figure 3 shows the growth rate (normalized to the real frequency) as a function of the alpha particle density scale length L_α with $\beta_\alpha = \beta_\alpha(0)\exp(-r^2/L_\alpha^2)$ for the different values of $\nu_{QL}^{(0)}/\omega_0$. The parameters chosen are the following: $a/R = 1/4$, $\rho_{\alpha 0}/a = 0.05$ with $\rho_{\alpha 0}$ the alpha Larmor radius, $v_{\alpha 0} = 2v_A$, $\beta_\alpha(0) = 1\%$, $T_e = 10$ keV, $n_e = 10^{14}$ cm^{-3} and $\Omega/k_{0\perp}v_{\alpha 0} = 2^{-1/2}$. It should be noted that $\omega_0 \sim v_A/3R$ while the alpha transit frequency $\omega_{t\alpha} \approx v_\alpha/qR \sim v_A/R$, so that the condition $\omega < \omega_{t\alpha}$, which was used in Section 2, is satisfied for the given set of parameters.

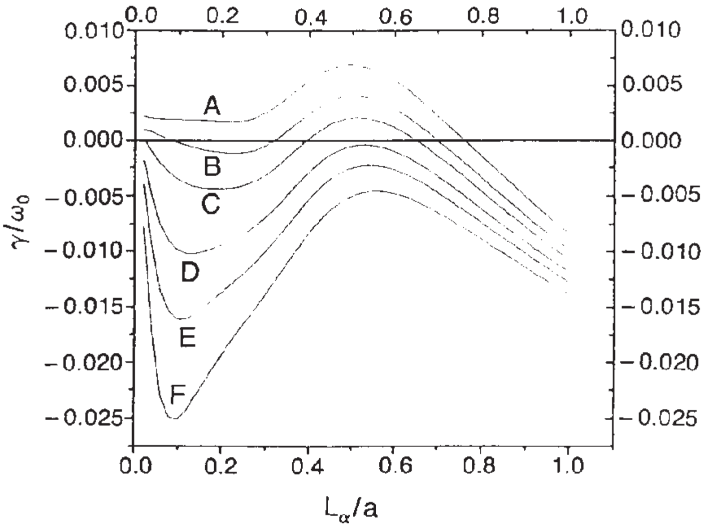

FIG. 3. Growth rate (normalized to the real frequency ω_0) for the n = −1 TAE mode as a function of the alpha particle density gradient scale length, for different values of $\nu_{QL}^{(0)}/\omega_0$: A, 0; B, 9×10^{-4}; C, 2×10^{-3}; D, 4×10^{-3}; E, 6×10^{-3}; F, 9×10^{-3}.

5. CONCLUSION

As one can see from Fig. 3, suppression of the instability occurs for $\nu_{QL}^{(0)}/\omega_0 \approx 5 \times 10^{-3}$, which is an order of magnitude agreement with an analytical estimation (see Appendix). With the definition of $\nu_{QL}^{(0)}$ and $\Omega/k_{0\perp}v_{\alpha 0} = 2^{-1/2}$ one could obtain for the RF field threshold amplitude

$$\{E\ [\mathrm{V/cm}]\}^2 \approx 10^{-7}(\omega_0\Omega)\ [\mathrm{Hz}^2]$$

which for $\omega_0 \sim 100$ kHz and $\Omega = 2.5$ GHz gives $E_{\mathrm{thr}} \sim 5$ kV/cm. The chosen value of $\Omega/k_{0\perp}$ corresponds to the refractive index $N_\perp \approx 32$, which for $N_\parallel \approx 1.6$ gives

$$E_\parallel \sim \frac{N_\parallel}{N_\perp} E_\perp \approx 250\ \mathrm{V/cm}$$

Then one could obtain for the required slow mode power flux

$$S_0 \sim \frac{cN_\perp}{8\pi} |E_\parallel|^2 \approx 4\ \mathrm{kW/cm^2}$$

which for an antenna with a surface area of $\sim 10^4$ cm^2 gives a total LH power $P_{\mathrm{LH}} \sim 40$ MW. This value is an order of magnitude lower than that needed for continuous operation of a reactor [15].

It should be noted that we use the simplest form of the equilibrium distribution F_α (18)–(20), which is valid if one neglects the prompt losses enhancement due to quasi-linear diffusion in an RF field. This problem was considered in Ref. [14], where it was shown that prompt losses enhancement is practically independent of the RF power because the height of the 'plateau' formed in the resonant part of velocity space does not depend on $\nu_{\mathrm{QL}}^{(0)}\tau_{\mathrm{sl}}$ for $\nu_{\mathrm{QL}}^{(0)}\tau_{\mathrm{sl}} \gg 1$.

The final remark is that the solution of the drift kinetic equation (7), (8) is valid in the zero orbit width approximation. The general solution of the linearized collisionless (and free from RF induced quasi-linear diffusion) drift kinetic equation for global MHD modes, which includes finite orbit width effects, was obtained recently in Ref. [16], where it was shown that the finite orbit width had a stabilizing effect on the TAE mode.

Appendix

To obtain an order of magnitude estimate of the suppression threshold, we first perform the spatial averaging of expression

$$H \equiv \left(\frac{\nu_{\mathrm{QL}}(\varepsilon,r)}{\omega}\right)^{1/2} \frac{1}{2 + \Delta_+(\varepsilon,r) + \Delta_-(\varepsilon,r)} \left[\frac{2}{\Delta_+}\left(1 - \frac{1}{\Delta_+^2}\right) + \left(\frac{1}{\Delta_-^2} - \frac{1}{\Delta_+^2}\right)\frac{\Delta_-}{\Delta_+} + \frac{2}{\Delta_-}\left(1 - \frac{1}{\Delta_-^2}\right) + \left(\frac{1}{\Delta_+^2} - \frac{1}{\Delta_-^2}\right)\frac{\Delta_+}{\Delta_-}\right]$$

which contributes to (12). Using approximation

$$\Delta_\pm^{m\pm1}(\varepsilon,r) = \left[1 \mp \frac{4\pi\,[m \pm 1 - nq(r)]}{q(r)} \ln^{-1}\left(\frac{256\omega}{\nu_{\mathrm{QL}}}\right) \frac{(\varepsilon r/R)^{1/2}}{\omega R}\right]^{1/2} \approx \left[1 \mp \left(\frac{r}{\delta_{m\pm1}}\right)^{1/2}\right]^{1/2} \tag{21}$$

and letting $x = (r/a)^{1/2}$, one could represent the spatial integral in the form

$$\left|\int_0^1 H\,\mathrm{d}x\right| \sim \left|\int_0^1 \frac{\mathrm{d}x}{x^{1/2}\left[2 + \left(1 + \frac{x}{\Delta}\right)^{1/2} + \left(1 - \frac{x}{\Delta}\right)^{1/2}\right]} \left(\frac{2}{\left(1 + \frac{x}{\Delta}\right)^{1/2}} + \frac{2}{\left(1 - \frac{x}{\Delta}\right)^{1/2}} - \frac{2}{\left(1 + \frac{x}{\Delta}\right)^{3/2}} - \frac{2}{\left(1 - \frac{x}{\Delta}\right)^{3/2}} + \frac{2}{\left(1 - \frac{x^2}{\Delta^2}\right)^{1/2}} - \frac{(1 - x/\Delta)^{1/2}}{\left(1 + \frac{x}{\Delta}\right)^{3/2}} - \frac{(1 + x/\Delta)^{1/2}}{\left(1 - \frac{x}{\Delta}\right)^{3/2}}\right)\right| \tag{22}$$

where

$$\Delta \sim \frac{1}{4\pi} \ln\left(\frac{256\omega}{\nu_{\mathrm{QL}}}\right) \frac{\omega R}{(\varepsilon a/R)^{1/2}}$$

The branch points in (21) occur for $x < 1$. Thus taking into account the fact that $\beta_\alpha \sim \exp(-x^4/L_\alpha^4)$, $L_\alpha < 1$, one could obtain an upper estimate for the suppression threshold by extending the upper integration limit in (22) to infinity, then observe that $|H| \sim |x^{-3/2}|$ as $x \to \infty$ and that the branch points in H occur at $x = 0$ and $x = \Delta$ which for $\mathrm{Im}\,\omega > 0$ is above the real x axis. As a result, the path of the x integration can be deformed to be down the imaginary x axis to obtain

$$\int_0^\infty H\,dx = \int_{-0i}^{-\infty i} H\,dx$$

Consequently, letting $iy = x/\Delta$ along the deformed path $x = i|x|$ gives

$$\left|\int_0^\infty H\,dx\right| = \left(\frac{\Delta}{2}\right)^{1/2}\int_0^\infty dy\,\frac{4y^2 + 2^{3/2}y(1+y^2)^{1/4}[2 + (1+y^2)^{1/2}][1 - (1+y^2)^{-1/2}]^{1/2}}{2\left[1 + \left(\frac{(1+y^2)^{1/2}}{2} + \frac{1}{2}\right)^{1/2}\right](1+y^2)^{3/2}y^{1/2}}$$

where $1 \pm iy = (1+y^2)^{1/2}\exp(\pm i\tan^{-1}y)$ has been used.

Letting $y = \sinh\varphi$ with $\exp(\varphi/2) = t$ gives

$$\left|\int_0^\infty dx\,H\right| =$$

$$4\Delta^{1/2}\left(\int_1^\infty \frac{(t^4-1)^{1/2}(t^2-1)}{(t+1)^2(t^4+1)}\,dt + 2\int_1^\infty \frac{(t^4-1)^{3/2}t}{(t+1)^2(t^4+1)^2}\,dt + 4\int_1^\infty \frac{(t^4-1)^{1/2}(t^2-1)t^2}{(t+1)^2(t^4+1)^2}\,dt\right) \approx 3\Delta^{1/2}$$

Thus, a rough estimation for the TAE growth rate is (the contribution from $K_{m\pm1}$ is the most pronounced)

$$\gamma \propto 1 - \left(\frac{\nu_{QL}}{\omega}\right)^{1/2}\ln\left(\frac{256\omega}{\nu_{QL}}\right)$$

which gives for the suppression threshold $\nu_{QL}/\omega \approx 10^{-2}$. This value agrees with the numerical result within a factor of 2.

ACKNOWLEDGEMENT

This work was partly supported by the G. Soros Foundation.

REFERENCES

[1] FU, G.Y., VAN DAM, Y.W., Phys. Fluids B **1** (1989) 1949.
[2] CHENG, C.Z., Phys. Fluids B **3** (1991) 2463.
[3] BETTI, R., FREIDBERG, Y.P., Phys. Fluids B **3** (1991) 538.
[4] BETTI, R., FREIDBERG, Y.P., Phys. Fluids B **4** (1992) 1465.
[5] HSU, C.T., SIGMAR, D.Y., Phys. Fluids B **4** (1992) 1492.
[6] BERK, H.L., et al., Phys. Fluids B **4** (1992) 1806.
[7] GORELENKOV, N.M., SHARAPOV, S.E., Phys. Scr. **45** (1992) 163.
[8] KARNEY, C.F.F., Phys. Fluids **22** (1979) 2188.
[9] DE ANDRADE, M.C.R., et al., Interaction of Lower Hybrid with Fast Ions in JET, Preprint JET-P(93)37, JET Joint Undertaking, Abingdon, Oxfordshire (1993) 79.
[10] MARCHENKO, V.S., Nucl. Fusion **33** (1993) 1871.
[11] CATTO, Y., ROSENBLUTH, M.N., Phys. Fluids **24** (1981) 243.
[12] MAZUR, V.A., MIKHAJLOVSKIJ, A.B., Nucl. Fusion **17** (1977) 193.
[13] LI, Y.M., et al., Phys. Fluids **30** (1987) 1466.
[14] BELIKOV, V.S., SILIVRA, O.A., Fiz. Plazmy **19** (1993) 947 (in Russian).
[15] REBUT, P.H., et al., The ITER Challenge, Preprint JET-P(92)92, JET Joint Undertaking, Abingdon, Oxfordshire (1992).
[16] PORCELLI, F., et al., Phys. Plasmas **1** (1994) 470.

(Manuscript received 1 August 1994
Final manuscript accepted 20 February 1995)